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Synthetic ΔK frequency plans in relation to radio wave object characterization

Eldar Aarholt

Environmental Surveillance Technology Programme PFM, Lilleström, Norway

Abstract. A system to generate data channels to enhance the classification of objects by means of selective radar frequency and antenna distribution plans using signal generating and processing methods which can be utilized in conventional multifrequency radar systems. The implementation of specific synthetic ΔK algorithms for the generation of large numbers of data channels derived from the initially selected radar frequency or antenna distribution plans increases the amount of information available for classification after detection. The distance to a reflector can be uniquely resolved across virtually any range ambiguity interval, and at the same time the reflector velocity can be determined over virtually any practical velocity variation.

1. General Introduction

For more than a decade the author has been working in a research group actively investigating the use of ΔK radar technology for various purposes, such as Ocean Spectral Analysis, Internal Wake Detection, Ground Penetrating Radar applications, and Non-Cooperative Target Recognition. Several radars have been built, and a large number of experiments have been conducted. The versatility of a ΔK radar system has been widely documented [*Gjessing*, 1986], and therefore this contribution will consider signal-generating techniques that will enhance the ΔK radar capabilities even further. Some basic calculations showing the ΔK radar implementation have also been included to make this contribution not entirely theoretical. Recent technological advances in systems using direct digital synthesizers, surface acoustic wave filters, and equipment for spectral analysis provide a target adaptive matched illumination capability with several degrees of freedom and an optimum signal-processing solution in the receiver.

The scattering processes of targets can be modeled in great detail using powerful computers, and this, combined with adaptive sensor technology, provides the user with a data assimilation capability when searching for very small scattering centers in an otherwise perturbed medium (clutter). In this context it is particularly important to distinguish between clutter and noise. The clutter spectral density is inherently related to the transmitted signal spectrum and may be highly correlated, while the background noise spectrum is independent of the transmitted signal and as such is independent of range [*DiFranco and Rubin*, 1968]. In a practical situation the background noise interference caused by internal thermal noise in amplifiers or external environmental noise from the sun, which imposes a limit on the minimum detectable signal, is usually much weaker than the clutter from unwanted backscattered signals from objects in the field of view, such as ocean waves, rain, or vegetation. The challenge in this context is to optimize the signal-to-clutter ratio and not necessarily that of the signal-to-noise.

When dealing with a stochastic process such as the sea surface, it can be characterized in terms of the spatial (and temporal) autocorrelation function $R_\sigma(\Delta\mathbf{r}, t)$ and the corresponding $\Delta\mathbf{K}$ spectrum

$$R(\Delta\mathbf{K}) \sim \int E(\mathbf{K})E^*(\mathbf{K}+\Delta\mathbf{K})\,d\mathbf{K} \sim \int R_\sigma(\Delta\mathbf{r})e^{-j\Delta\mathbf{K}\cdot\Delta\mathbf{r}}\,d\Delta\mathbf{r} \quad (1)$$

It is evident that any a priori information of the object and the background clutter would simplify the classification problem, especially in the case where the object possesses features (see Figure 1) distinctly different from the background. Modeling and experimental tests are important factors in this context when analyzing the object using several signature domains to extract the specific features of interest. The implication of this is that it is possible to reduce

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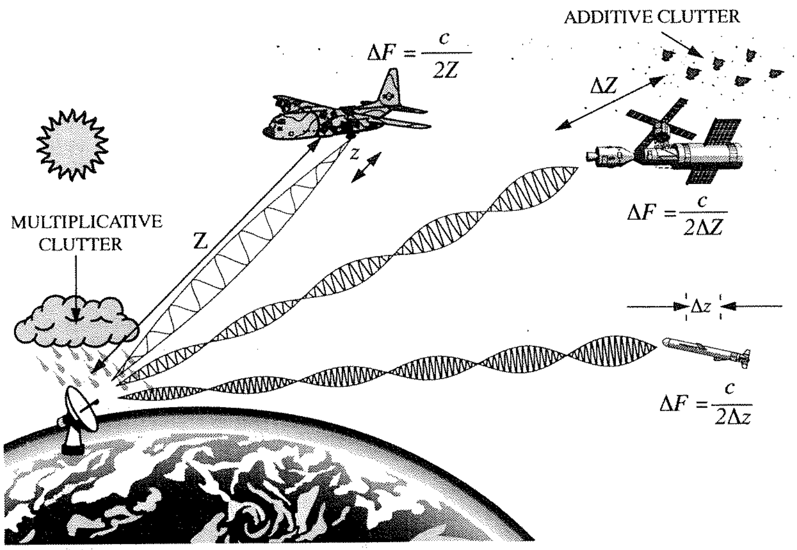

Figure 1. The distance to one scattering object, Z, can be measured by recording the phase of the beat frequency ΔF from two carriers. The absolute phase accuracy of the phase detectors determines the positioning accuracy, z, of the system. The size of an object can be determined by transmitting two carriers separated in frequency so that half the wavelength of the difference frequency, ΔF, equals the size of the object, ΔZ. The shape of the object, Δz, is obtained by transmitting a number of carriers of greater frequency separation, that is, increasing the bandwidth of the system, ΔF, and then comparing the return for each of the transmitted frequencies. Any a priori information about the object of interest, the propagation medium, and the background should be fed into the illumination processor to optimize the detection-identification capability.

the bandwidth of a radar system if sufficient a priori information about the object is known.

The following sections rely on the solution of the basic wave equations using the Born (volume scattering) and the Kirchoff (surface scattering) approximations. The characterization of a scattering object in four dimensions (three in space and one in time) is illustrated with some examples demonstrating the matched illumination concept of which the ΔK methods are a subset.

If the scatterers are distributed in space, the field strength of the backscattered wave is a function of the wavenumber (frequency), that is, the Fourier transform of the delay function. Therefore, if the received field strength is measured as a function of frequency, the spatial distribution of the object can be found using an inverse Fourier transform.

A natural extension of the multifrequency ΔK spaced frequency concept (synthetic pulse) is to apply the method to antenna systems (synthetic antenna distribution), as seen in Figure 2, where the resolution in range, Δz, is found as

$$\Delta z = c/2\Delta F_{\max} \tag{2}$$

and the ambiguity range interval, z_0, equals

$$z_0 = c/2\Delta F_{\min} \tag{3}$$

Similarly, the cross-range resolution, Δx, in a synthetic antenna system equals

$$\Delta x = CR/FD \tag{4}$$

and the cross-range ambiguity interval, x_0, equals

$$x_0 = CR/Fd \tag{5}$$

The ΔK radar is constructed by transmitting two or more carrier frequencies (K channels), $V_F(t)$, sampling the coherent baseband I and Q components of the radar return of all carriers, and combining the received channels by multiplying all channels by the complex conjugate of all the other channels, such that

$$V_{\Delta F_m}(t) = V_{F_i}(t)V^*_{F_j}(t) \tag{6}$$

$$i = 1, n-1 \quad j = i+1, n \quad m = 1, [n(n-1)/2]$$

The number and distribution of the ΔK channels depend on the K frequency plan initially used. Preferably, the frequency plan should distribute the ΔK channels evenly in unit steps across the transmitted bandwidth. This can be accomplished using a mathematical technique described by *Robinson and Bernstein* [1967]. However, using five or more discrete carriers will always give undesirable "holes" in the resulting ΔK frequency plan. This technique will now be taken one step further, first, by a careful selection of the frequency plan and second, by generating higher-order ΔK derivatives, termed super ΔK ($S\Delta K$) channels, by combining three, four, or more K channels. As in most instances, some limitations do exist. These are also considered.

2. Optimal Methods for Object Characterization

The distance to an object [*Gjessing*, 1981] can be uniquely resolved by measuring the phase difference between two carriers, F_1 and F_2. Consider one scattering object at a distance of $Z = 100$ km. Constructing a beat channel, ΔF, of frequency 1.5 kHz (wavelength = 200 km) gives a range interval of 100 km. If the measurement system has a phase accuracy of 1°, the maximum positioning resolution would be $z = 100$ km/360 = 278 m. If absolute range, Z, is not required, the size of the object, ΔZ, can be determined by transmitting two carriers separated in frequency so that half the difference frequency wavelength equals the size of the object, that is, the beat frequency wavelength couples to the size of the object.

By increasing the radar carrier bandwidth the shape of the object (range profile) can be determined. Transmitting a number of carriers simultaneously will give information about the geometric distribution and size of the scattering centers of the object. The maximum resolution, Δz, is obtained using a bandwidth ΔF such that $\Delta F = c/2\Delta z$. These examples are illustrated in Figure 1.

Figure 3 shows typical ΔK channel time series return from two different aircraft. Only the real part is shown. The radar used for these measurements operated at 10 GHz using six coherent carriers. Because of the carrier frequency distribution, certain ΔK channels cannot be constructed, that is, channels 14 and 15 are not available. This K channel distribution method commonly used gives a finite number of possible ΔK channel combinations. Six K channels are used to generate the 15 ΔK channels shown.

The spectral relationship between the received ΔK channels contains information of the object's reflector distribution (i.e., the distribution of scattering centers), as seen in Figure 4, in that the ΔK channels couple to and give a higher return when the distance between two reflectors equals that of half the ΔK channel wavelength. The ΔK spectral features, such as the amplitude relationship between channels seen in Figure 4, can be used for classification purposes [*Aarholt*, 1989].

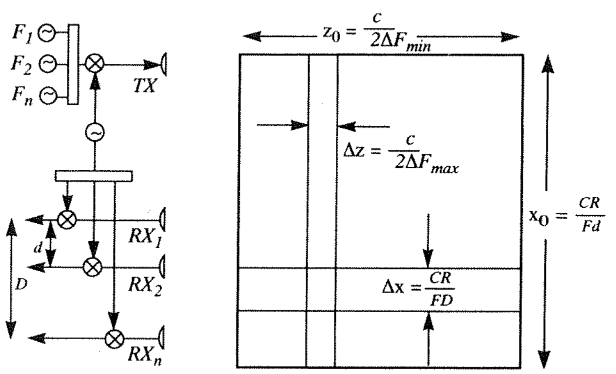

Figure 2. Range resolution, Δz, is related to the maximum radar bandwidth used in the system, $\Delta F_{\max}$, and the range ambiguity interval, z_0, is determined by the minimum frequency separation, $\Delta F_{\min}$. The cross-range resolution, Δx, is similarly determined by the maximum antenna spacing, D, and the cross-range ambiguity interval, x_0, is related to the minimum antenna spacing, d.

When computing the inverse fast Fourier transform (IFFT) across the received ΔK channel samples, the range profile of the object can be determined. An important factor in this context is the limited number of frequencies which can be transmitted simultaneously (N_F) and the low dynamic range [$20\log(n_{\Delta F})$] obtained in the inversion, where n equals the number of ΔK channels. If the number of ΔK channels could be increased without changing the radar hardware, an improved dynamic range in the range profile computations would result, as well as an increased sampling across the radar bandwidth. This will be shown in the next sections.

3. Multifrequency Radar Systems

3.1. Radar Frequency Plans

A radar's intermediate frequency channels are upconverted to a suitable carrier, such as 10 GHz, and the number of intermediate frequencies are distributed across the radar bandwidth. Typical bandwidths can be from 10 MHz to 1000 MHz. A typical ΔK

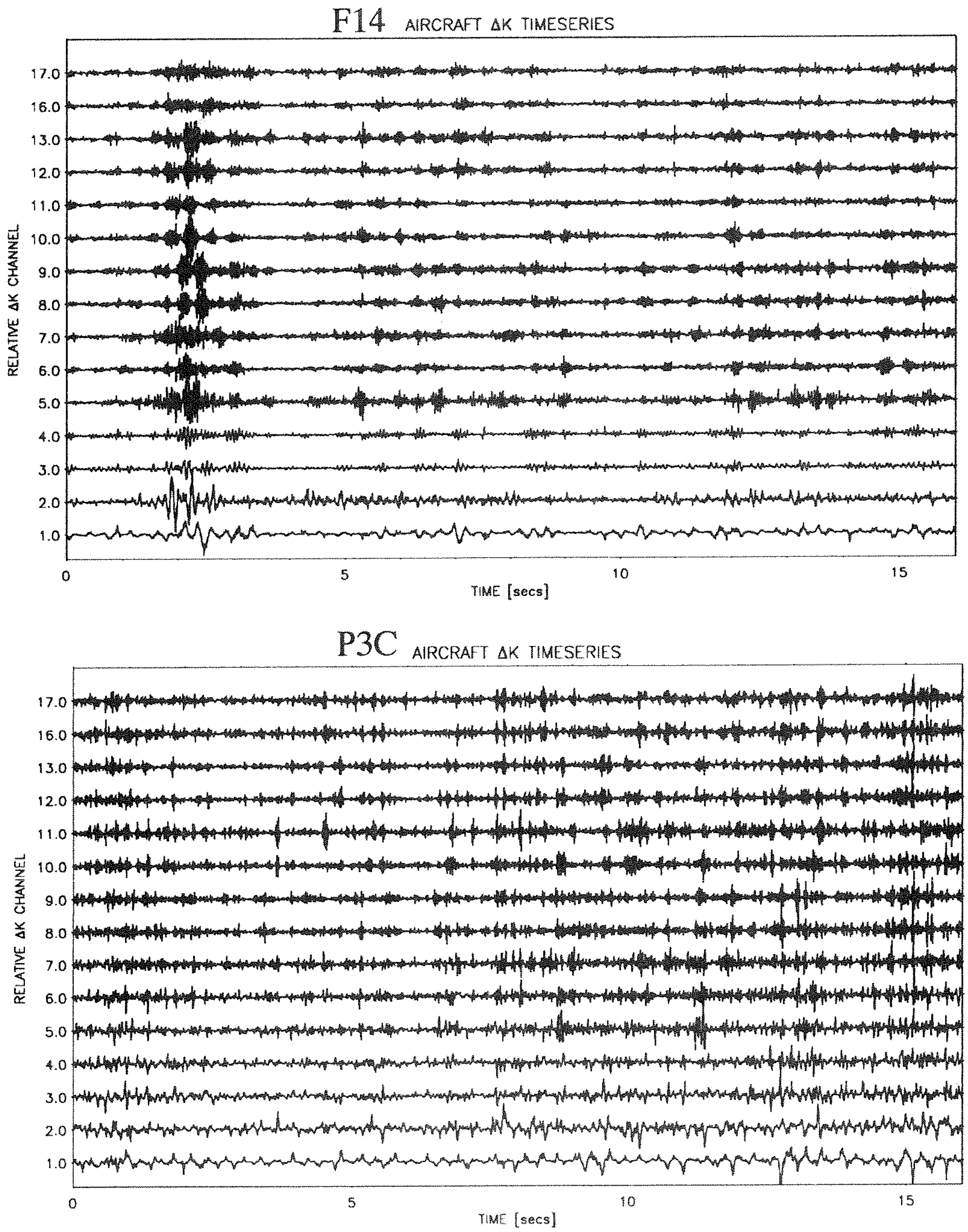

Figure 3. An example of the ΔK channel time series from two different aircraft, the F14 and the P3C. Note the increasing time series frequency as a function of ΔK channel frequency and the amplitude modulation unique to each channel.

radar system uses six unevenly distributed coherent carriers ($n = 6$) which gives 15 ΔK channels [$m = n(n-1)/2$]. An added feature of the uneven carrier distribution is that intermodulation products will be reduced and the need for backing off the power equally reduced.

The frequency plan is specified using an incremental frequency selection plan. The first K channel value

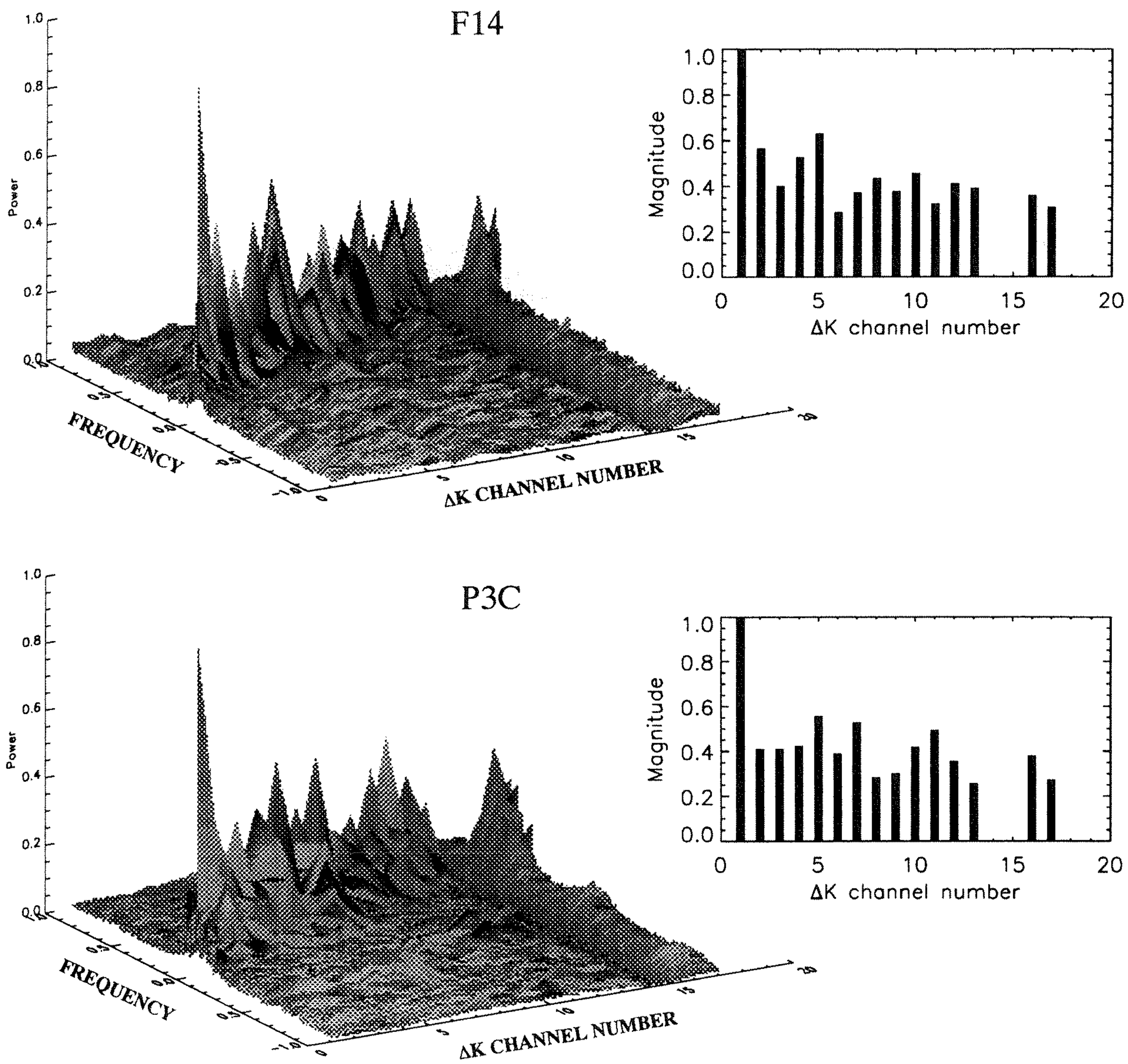

Figure 4. ΔK spectral feature presentation of two different aircraft, the F14 and the P3C, and the maximum return for each ΔK channel. The amplitude relationship between the received ΔK channels contains information of the distribution of an object's scattering centers. The ΔK channels couple to the distance between different reflectors equal to half the ΔK channel wavelength. The spectral features are descriptive of the object and can be used for classification purposes.

is always zero. By adding each element in the frequency plan the selected frequency combination is generated. The resulting channels can be translated into any frequency band, since waveform performance is only dependent on the difference frequency between the carriers. Also, any identical preprocessing modulation introduced to all carriers during signal generation will be removed when the ΔK channels are generated. Using a five-element (1, 3, 6, 2, 5) incremental frequency plan [*Robinson and Bernstein*, 1967], adding the numbers gives the resulting K channel distribution, as shown in Table 1, and by combining these channels the resulting equally spaced ΔK channels in Table 2 are generated.

To use the frequency plan in a radar system (see the appendix), the lowest intermediate frequency and

Table 1. Six K Channel Distribution

Channel Number	K Channel Frequency
K_1	0
K_2	1
K_3	4
K_4	10
K_5	12
K_6	17

Table is based on a distributed incremental frequency plan of (1, 3, 6, 2, 5) resulting in equal spacing at ΔK level as seen in Table 2. The K channel frequency values are found by adding the elements in the incremental frequency plan.

the radar bandwidth to be used must be determined. The lowest K channel is then placed at K_1 and the highest K channel at K_6. The other channels are distributed in between according to the K channel distribution in Table 1. It is seen from Table 2 that ΔK channel frequency samples 14 and 15 are missing resulting in holes in the ΔK frequency table.

3.2. $S\Delta K$ Channel Generation

The purpose of the following is to eliminate the holes in the frequency tables used in, for example, ΔK radar systems and at the same time increase the number of equally spaced ΔK channels. Using the five-element incremental frequency example in section 3.1, the maximum number of ΔK channels would be 15. By permutating these 15 ΔK channels once

Table 2. Fifteen ΔK Channel Distribution

Channel Number	K Channel Combination	ΔK Channel Frequency
ΔK_1	$K_1K_2^*$	1
ΔK_2	$K_4K_5^*$	2
ΔK_3	$K_2K_3^*$	3
ΔK_4	$K_1K_3^*$	4
ΔK_5	$K_5K_6^*$	5
ΔK_6	$K_3K_4^*$	6
ΔK_7	$K_4K_6^*$	7
ΔK_8	$K_3K_5^*$	8
ΔK_9	$K_2K_4^*$	9
ΔK_{10}	$K_1K_4^*$	10
ΔK_{11}	$K_2K_5^*$	11
ΔK_{12}	$K_1K_5^*$	12
ΔK_{13}	$K_3K_6^*$	13
ΔK_{14}	$K_2K_6^*$	16
ΔK_{15}	$K_1K_6^*$	17

Table is based on Table 1 and gives equal spacing between frequencies. The ΔK channels are computed by multiplying all K channels by the complex conjugate of all the other K channels, that is, ΔK_{10} = signal(K_1)conj[signal(K_4)]. Two undesirable "holes" in the frequency plan are seen between ΔK channel numbers 13 and 14.

Table 3. Six K Channel Frequency Distribution

Channel Number	K Channel Frequency
K_1	0
K_2	4
K_3	16
K_4	30
K_5	45
K_6	54

Table originates from the incremental frequency selection plan (4, 12, 14, 15, 9) and results in 46 equally spaced $S\Delta K$ channels, as seen in Table 4, the first 42 without holes. The K channel frequency values are found by adding the elements in the incremental frequency plan.

more to a higher-order ΔK level an additional 105 $S\Delta K$ channels will be generated [$k = m(m-1)/2$]. This is equivalent to multiplying and permutating sets of four K channels, that is,

$$V_{\Delta\Delta F_k}(t) = V_{\Delta F_i}(t)V^*{}_{\Delta F_j}(t) \tag{7}$$

$$i = 1, m-1 \quad j = i+1, m \quad k = 1, [m(m-1)/2]$$

Additional $S\Delta K$ channels can be obtained by multiplying and permutating sets of three K channels. This is equivalent to multiplying and permutating the first set of 15 ΔK channels by each K channel again, giving 90 more combinations.

These computations result in a total of 210 possible ΔK channels. Depending on the initial frequency plan, the number of unique $S\Delta K$ channels will vary significantly ($m < N < 210$). In the case of the example in Table 1 a total of only 17 unique $S\Delta K$ channels are generated. In practice, it is not possible to avoid some ΔK channel replication. However, by careful selection of the original frequency plan, channel replication can be minimized and the number of holes in the plan virtually eliminated.

An example of such an $S\Delta K$ incremental frequency selection plan is (4, 12, 14, 15, 9), which in turn generates the K channel frequency plan seen in Table 3 and subsequently the $S\Delta K$ channels seen in Table 4.

Table 4. $S\Delta K$ Channel Distribution

Channel Number	$S\Delta K$ Channel Frequency
1:42	1:42
43:46	45:46 50 54

A total of 46 equally spaced $S\Delta K$ channels are generated using only six K channels. Holes exist only at frequency positions above 42.

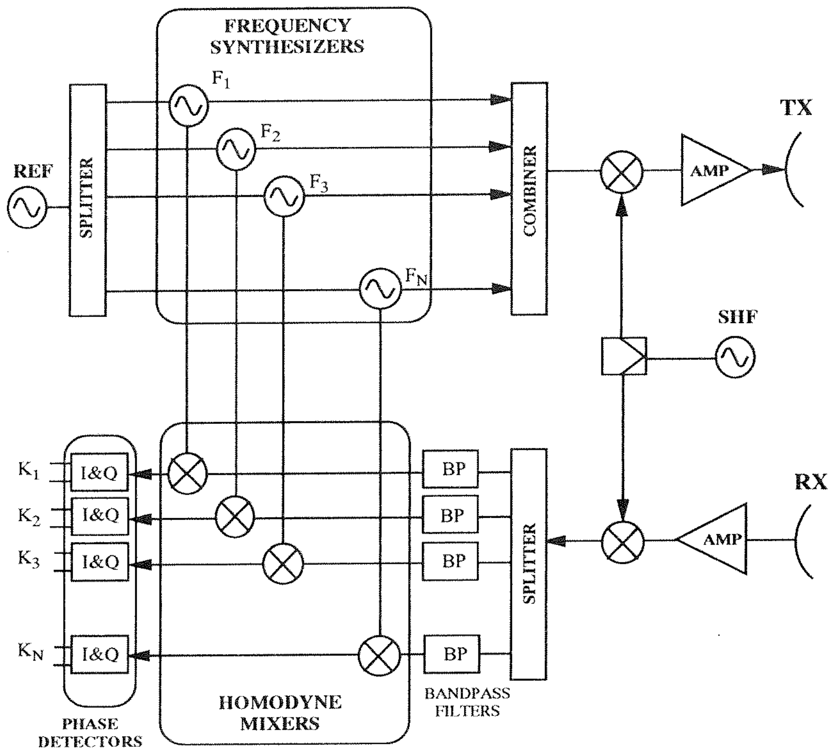

Figure 5a. A typical multifrequency ΔK radar system for use with the synthetic ΔK method. The received signals, $K_1 - K_n$, are connected to the synthetic signal processing unit (Figure 5b).

Please observe that the K channel frequencies are relative values based upon the frequency and the bandwidth used in any radar system implementation.

The incremental frequency selection plan giving the $S\Delta K$ channel distribution seen in Table 4 is generated by optimizing the number of $S\Delta K$ channels and at the same time minimizing the highest $S\Delta K$ value. Using a few carriers (e.g., six) in a typical multifrequency radar system (see Figure 5a), a total of 46 equally spaced ΔK channels are generated, the first 42 without holes. An analog representation of a four-channel synthetic ΔK signal processing unit is seen in Figure 5b.

Several applications of ΔK radar systems are described by *Gjessing* [1986]. The radar bandwidth (ΔF) describes the smallest feature ($z_{\min}$) the ΔK radar channel will resolve on an object such that $\Delta F \geq c/2z_{\min}$, where c is speed of light, and the lowest ΔK frequency $\Delta F \leq c/2z_{\max}$ determines the largest object of interest ($z_{\max}$) which also equals the range ambiguity interval of the radar system. The introduction of the $S\Delta K$ channel allocation technique obviously does not change the radar bandwidth, but it dramatically increases the number of samples or ΔK channels across the bandwidth. These may be used for a ΔK matching scheme, which makes use of the fact that when two signals of slightly different frequency are multiplied together, the resulting beat (amplitude modulation) has a wavelength that equals that of the difference frequency. The increased number of $S\Delta K$ channels are therefore available to couple to an equally increased number of possible reflector combinations on the object. The synthetic ΔK technique also fills in the holes in the frequency plan, such as the ones described in section 3.1.

The effect of increasing the number of range bins is seen in Figure 6. A calculation of the range profile based on six equally spaced K channels (1, 2, 3, 4, 5, 6) using a rectangular window function is seen in Figure 6a, while distributing the six frequencies randomly across the bandwidth (1, 3, 7, 8, 11, 17) produces the range profile in Figure 6b.

An optimized ΔK frequency plan based on the incremental frequency plan (1, 3, 6, 2, 5) gives the return in Figure 6c. A greatly improved range profile computation is presented in Figure 6d, which is based

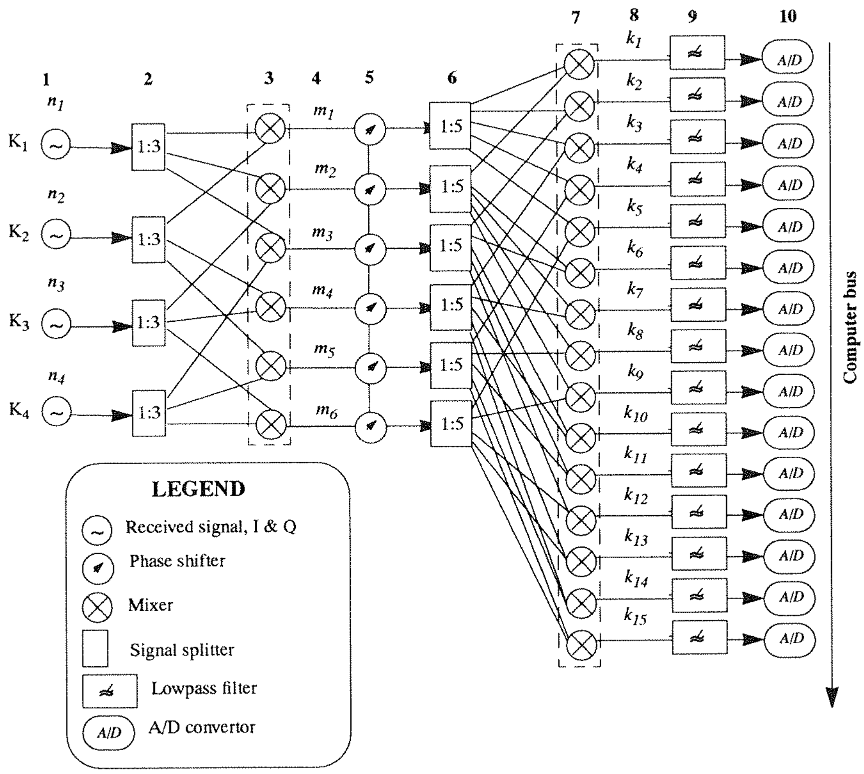

Figure 5b. An analog representation of a four-channel synthetic ΔK signal-processing unit. The (1) homodyned input, K_1–K_4, of the received $n = 4$ channels are each (2) split into $(n - 1)$ signals which are (3) multiplied with the complex conjugate of all other channels. The (4) phase of the resulting m ΔK channels $[m = n(n - 1)/2]$ are (5) calibrated with a reference phase. Each channel is then (6) split into $(m - 1)$ signals which are (7) multiplied with all the other channels, resulting in (8) a total of k $S\Delta K$ channels $[k = m(m - 1)/2]$. These channels are subsequently (9) low-pass filtered and (10) digitized for further processing.

on the synthetic ΔK optimized frequency plan (4, 12, 14, 15, 9) minimizing the number of holes. The first 42 frequency samples (without holes) are used in the inversion. This method gives an optimal use of the available radar carrier bandwidth by synthesizing numerous channels across the bandwidth.

3.3. Antenna Distribution Plans

The effect of the higher order of ΔK theory can also be applied to antenna applications such as radio astronomy [*Bracewell*, 1966], where the size of the antenna aperture may be a limiting factor. Combining several antennas to obtain low sidelobes and high directivity is commonly used for all electromagnetic applications, as seen in Figure 7.

A number of evenly spaced antennas result in a main beamwidth equal to the wavelength divided by the total antenna aperture, as seen in Figure 8a, such that

$$F(x,\phi) = \left| \sum_{i=0}^{n-1} A_i e^{-jkx_i \sin(\phi)} \right|^2 \qquad (8)$$

where $k = 2\pi f/c$, f is carrier frequency, c is speed of light, x is the position of the antenna element as a function of λ, and ϕ is the angle off-axis. The equivalent frequency domain relation can be computed by inserting a unit value into the first n real array elements of a complex data array (corresponding to n in-phase antenna elements in a rectangular aperture) and subsequently computing the fast Fourier transform (FFT), as shown in Figure 6.

The random thinning of antenna elements radically

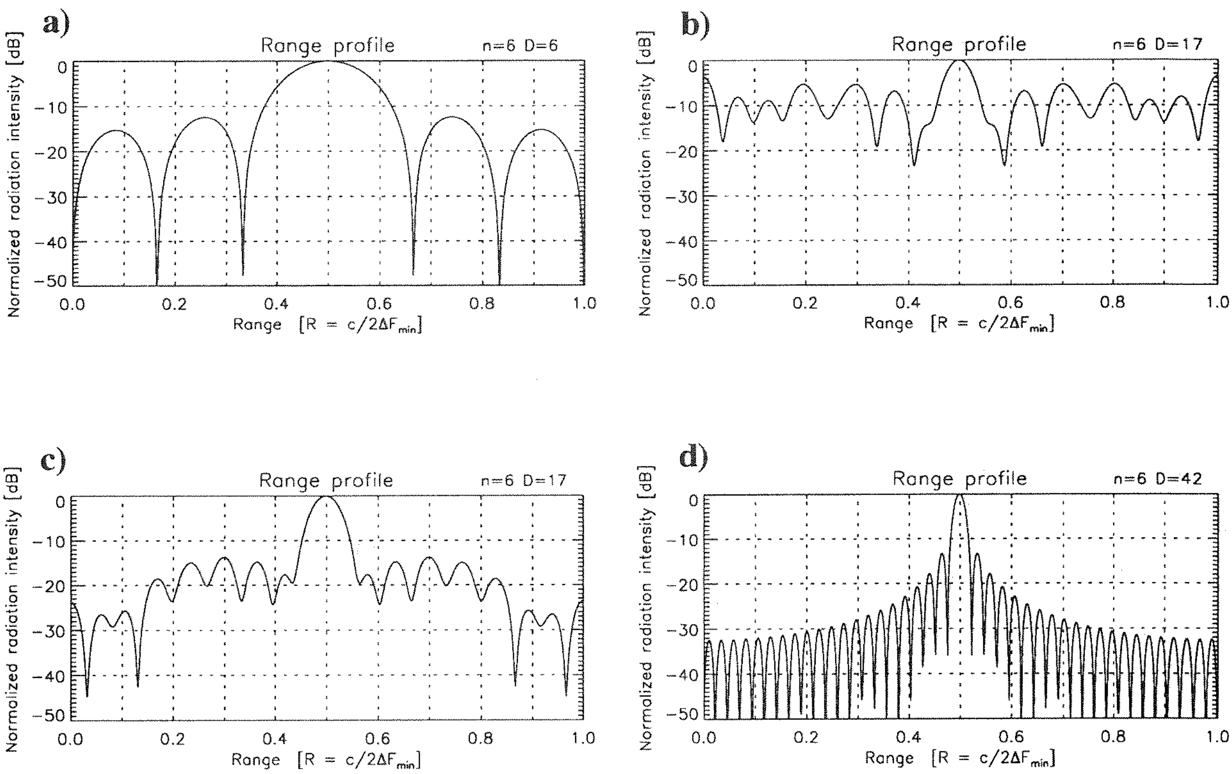

Figure 6. Range profile computations based on the (a) six equally spaced K frequencies (1, 2, 3, 4, 5, 6), (b) a randomly thinned six K frequency distribution (1, 3, 7, 8, 11, 17), (c) an optimized ΔK frequency plan (1, 3, 6, 2, 5), and (d) a synthetic ΔK frequency plan optimized for minimum number of holes (4, 12, 14, 15, 9). The synthetic ΔK frequency plan dramatically increases the dynamic range in the computations.

changes the radiation pattern of an antenna array. The effect of thinning the array by redistributing the six antennas used in the example in Figure 8a is shown in Figure 8b, where the six antenna elements are distributed across an aperture of 17 antenna positions separated by $\lambda/2$. The radiation pattern of such an antenna array is not easy to utilize, since the main lobe may only be about 5 dB above the sidelobes levels.

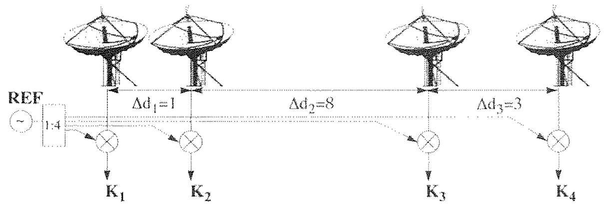

Figure 7. Antenna distribution plan for four antennae (RX_1–RX_4) distributed with a relative distance of Δd (1, 8, 3). The received signals from all antennas are demodulated to baseband, K_1–K_4, and are subsequently input to the synthetic signal processing unit (Figure 5b).

The reason for using 17 antenna positions becomes evident when considering means of synthesizing antenna arrays by combining the return from different antenna positions. To some extent this method fills in the holes between the active antenna elements in the array applying the ΔK technique described in section 3.1. The six antenna positions ($n = 6$) in Figure 8a can be redistributed to give 15 unique unit spaced Δd antenna spacings [$m = n(n - 1)/2$].

Separating the six antennas in the above example according to the incremental antenna position plan (1, 3, 6, 2, 5) results in the six antenna positions in Table 5 which are found by adding the elements in the incremental plan. Subsequently, the 15 synthesized antenna positions in Table 6 are generated by permutating the elements in the antenna position Table 5. The antenna diagram based on the Δd distributed

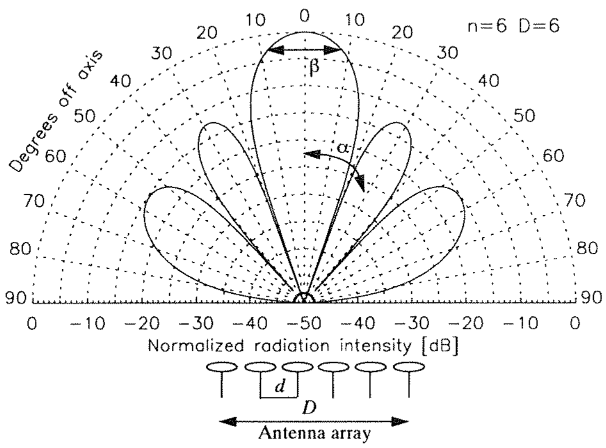

Figure 8a. The radiation pattern of a six-element equally spaced antenna array (1, 2, 3, 4, 5, 6). The combined aperture of the antennas (D) and the radio wavelength (λ) give an angular resolution $\beta = \lambda/D$. The antenna separation ($d = \lambda/2$) gives the angular distance to the nearest sidelobe, $\alpha = \lambda/d$. The geometric sum of the contribution of all antenna elements are added and plotted as a function of angle.

antenna position is seen in Figure 8c with sidelobes down about 15 dB as compared to the main lobe.

The radiation pattern of the antenna array can be improved further by applying the $S\Delta K$ frequency method described in section 3.2 to the antenna distribution. Figure 8d shows the synthesized antenna radiation pattern based on the incremental antenna spacing plan (4, 12, 14, 15, 9). Of the 46 unit spaced antenna elements, the first 42 are without holes, resulting in a greatly improved radiation pattern as compared to the examples above.

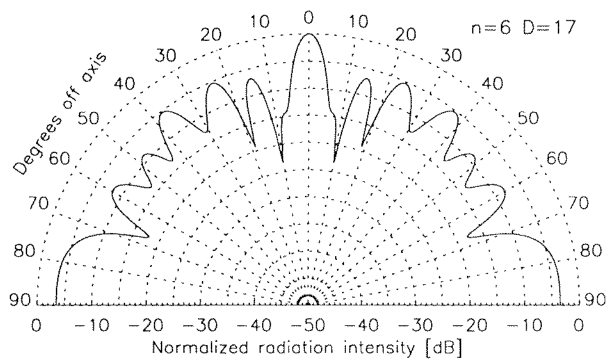

Figure 8b. The radiation pattern of a six-element antenna array distributed across an aperture of 17 antenna spacings of $\lambda/2$. The geometric sum of the contribution of all antenna elements are added and plotted as a function of angle. The antennas' positions are (1, 3, 7, 8, 11, 17) units of $\lambda/2$.

Table 5. Six Antenna Positions

Antenna Number	Antenna Position
d_1	0
d_2	1
d_3	4
d_4	10
d_5	12
d_6	17

Table is based on a distributed incremental position plan of (1, 3, 6, 2, 5) resulting in 15 equally spaced synthesized antenna spacings, as seen in Table 6.

4. Image Enhancement Using the Synthetic ΔK Method

Applying the synthetic ΔK method to a radar system greatly enhances the image in a radar scene. The method applied to the transmitted radar frequency combination results in a longer range interval as well as increased number of samples across the range interval. Applied to the antenna distribution, an improved sampling cross range can be obtained. The four range profile examples shown in Figure 6 can be combined with the antenna radiation pattern examples in Figure 8a through Figure 8d and ex-

Table 6. Fifteen Δd Antenna Distributions

Antenna Number	Antenna Combination	Δd Antenna Position
Δd_1	$d_1d_2^*$	1
Δd_2	$d_4d_5^*$	2
Δd_3	$d_2d_3^*$	3
Δd_4	$d_1d_3^*$	4
Δd_5	$d_5d_6^*$	5
Δd_6	$d_3d_4^*$	6
Δd_7	$d_4d_6^*$	7
Δd_8	$d_3d_5^*$	8
Δd_9	$d_2d_4^*$	9
Δd_{10}	$d_1d_4^*$	10
Δd_{11}	$d_2d_5^*$	11
Δd_{12}	$d_1d_5^*$	12
Δd_{13}	$d_3d_6^*$	13
Δd_{14}	$d_2d_6^*$	16
Δd_{15}	$d_1d_6^*$	17

Table is based on Table 5 and gives equal spacing between the synthesized antennas. The Δd antenna signals are computed by multiplying the signals from all d antenna positions by the complex conjugate signal from all the other antenna: that is, Δd_{10} = signal(d_1)conj[signal(d_4)]. Two undesirable holes in the antenna plan are seen between Δd positions 13 and 14, which will contribute to increased sidelobe levels. This is illustrated in Figure 8c.

tended to include cross-range–downrange resolution using six radar frequencies in the transmitter and six separate receiver antennas distributed cross-range. The result of the four different distribution methods is seen in Figures 9a–9d. The downrange and cross-range wavenumber coverage is greatly improved using the synthetic ΔK method, as seen in Figure 9d. Note the increased axis range for the $S\Delta K$ example.

4.1. Signal Processing Gain-Holographic Implementation

Computing the two-dimensional FFT based on the two-dimensional frequency and antenna matrix deduced from the images presented in Figure 9, the signal processing gain is seen in Figure 10 for the four different distribution plans: six equally spaced frequencies and antennas, a randomly thinned six-frequency and antenna distribution, an optimized ΔK frequency and antenna plan, and an $S\Delta K$ optimized frequency and antenna distribution. The synthetic ΔK method gives a greatly improved signal processing gain as compared with other methods.

The downrange and cross-range resolutions equal $\Delta z_{\mathrm{DR}} = c/2\Delta F_{\max}$ and $\Delta x_{\mathrm{CR}} = \Delta D_{\max}F/Rc$, respectively, and the corresponding downrange and cross-range intervals are $z_{0\mathrm{DR}} = c/2\Delta F_{\min}$ and $x_{0\mathrm{CR}} = \Delta D_{\min}F/Rc$.

Target inversion and range profiling are derived by computing the IFFT across the ΔK channel amplitude and phase in the frequency domain for every sample in the time domain. Table 7 lists the improved signal processing gain as a function of the number of K channels based on the conventional ΔK method, as compared to the new synthetic ΔK method.

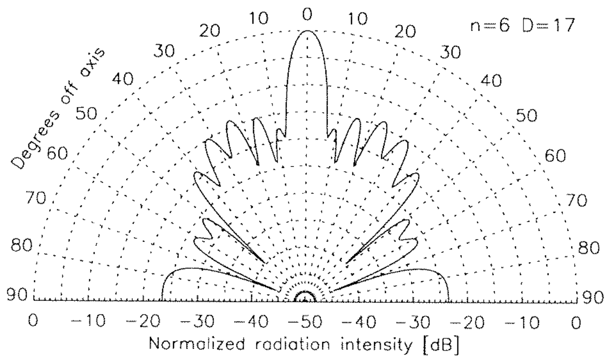

Figure 8c. The radiation pattern of a six-element antenna array distributed across an aperture of 17 antenna spacings of $\lambda/2$ using the incremental antenna spacing plan of (1, 3, 6, 2, 5) results in a synthesized 15-element antenna array. Two holes in the synthesized antenna distribution exist at positions 14 and 15.

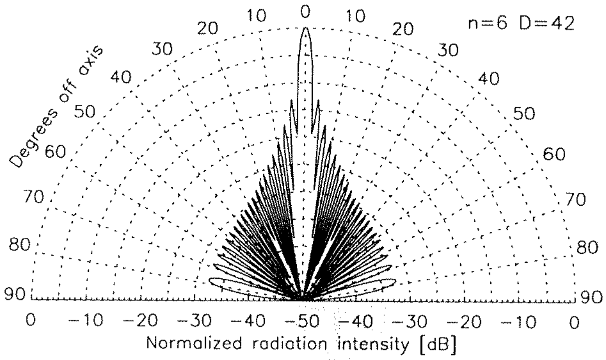

Figure 8d. The radiation pattern of a six-element antenna array distributed using the Synthetic ΔK method results in an aperture of 42 equally spaced ($\lambda/2$) antenna elements. The incremental antenna spacing plan used is (4, 12, 14, 15, 9). The number of synthesized antenna elements becomes 46. However, the first 42 elements are without holes, and only these are used in the computations to minimize sidelobe levels.

For range profile analysis an equal ΔK frequency separation layout in the frequency plan would be preferred. Because of the increased number of ΔK channels using the synthetic ΔK method, the dynamic range in the inversion is increased. For systems using larger numbers of K channels or other frequency plans optimized for maximum number of channels the processing gain is improved accordingly.

4.2. Synthetic ΔK Level Manipulation

The synthetic ΔK technique described can be applied to even further levels of ΔK processing. Combining the $S\Delta K$ channels once more results in several thousand possible ΔK channel combinations. About 20% of these will consist of unique ΔK channels if the original frequency plan is selected with care. Therefore an extremely detailed object description can be constructed. The synthetic ΔK method implies also that only a small number of carriers ($n \sim 8$) would be required to generate sufficient ΔK channels needed for most applications with a very powerful ΔK matching and signal-processing capability preserved in the system. Each new $S\Delta K$ level generation degrades the signal-to-noise ratio (S/N) by 3 dB, but if the S/N is sufficiently large at each level, the overall performance is not seriously degraded. The system also allows for bandwidth reduction after the final $S\Delta K$ generation.

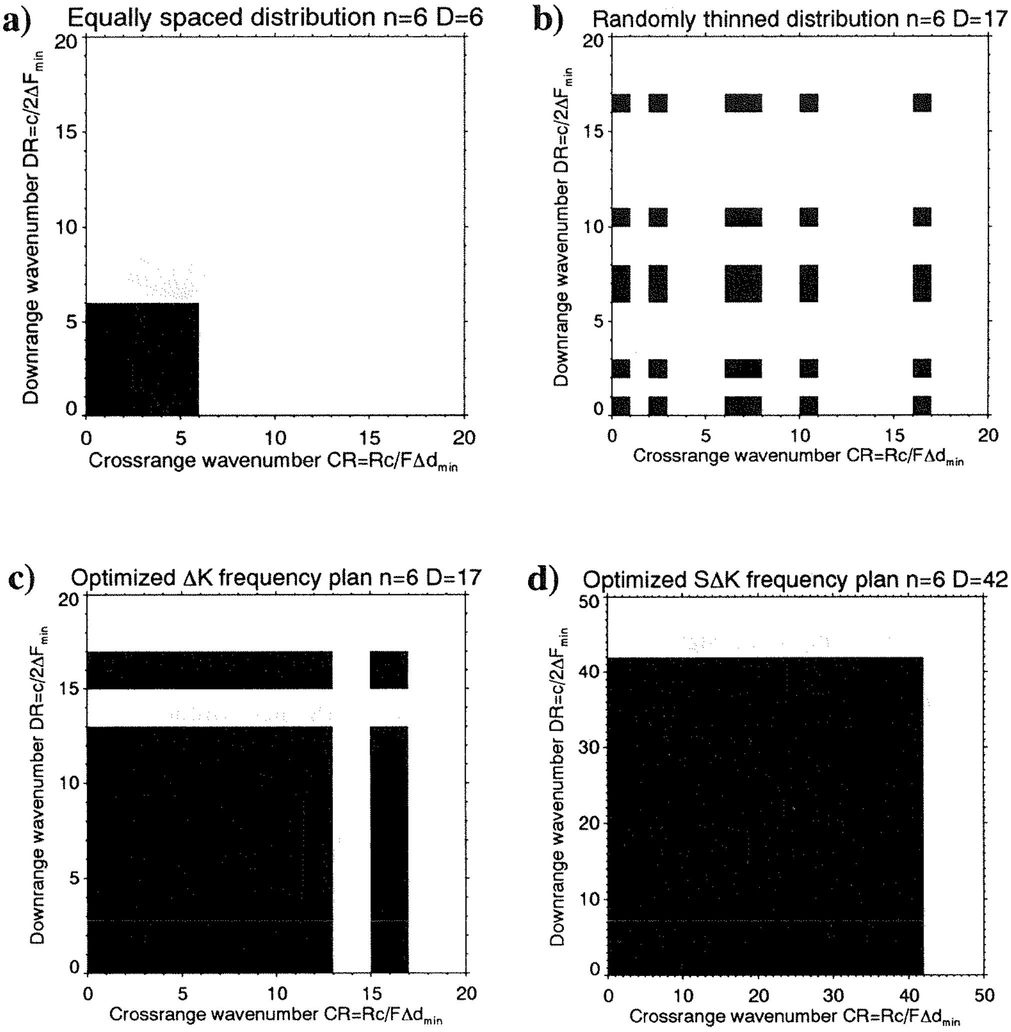

Figure 9. Downrange and cross-range wavenumber coverage using (a) six equally spaced frequencies and antennas, (b) a randomly thinned six-K frequency and antenna distribution, (c) an optimized ΔK frequency and antenna plan, and (d) an $S\Delta K$ optimized frequency and antenna plan. Note the increased axis range for the $S\Delta K$ example.

4.3. Filter Construction

The range intervals used in a conventional ΔK radar configuration can be about 200 m, as deduced from the lowest ΔK channel of typically 0.75 MHz. A limitation in this context is the finite bandwidth of the bandpass filters separating the K channels before downconversion to baseband for amplitude and phase detection. Narrow filters disguised as low-frequency beat channels of less than 1 kHz can easily be constructed using the synthetic ΔK technique giving range intervals of more than 150 km and at the same time maintaining bandwidths in the MHz range.

In practice, this means that the distance to a reflector can be uniquely resolved across virtually any range ambiguity interval, and the object's velocity can be determined over virtually any practical velocity variation using a ΔK system based on the synthetic ΔK method rather than the conventional system using first-level ΔK only.

A range interval of 150 km is constructed using an incremental frequency plan of (1, 0.999). The K channel frequency plan then becomes (0, 1.0, 1.999). The resulting ΔK frequency plan equals (0.999, 1.0, 1.999) and the $S\Delta K$ channel frequencies becomes (0.001, 0.999, 1.000).

The calculations are as follows: $Z = c/2\Delta F$, where

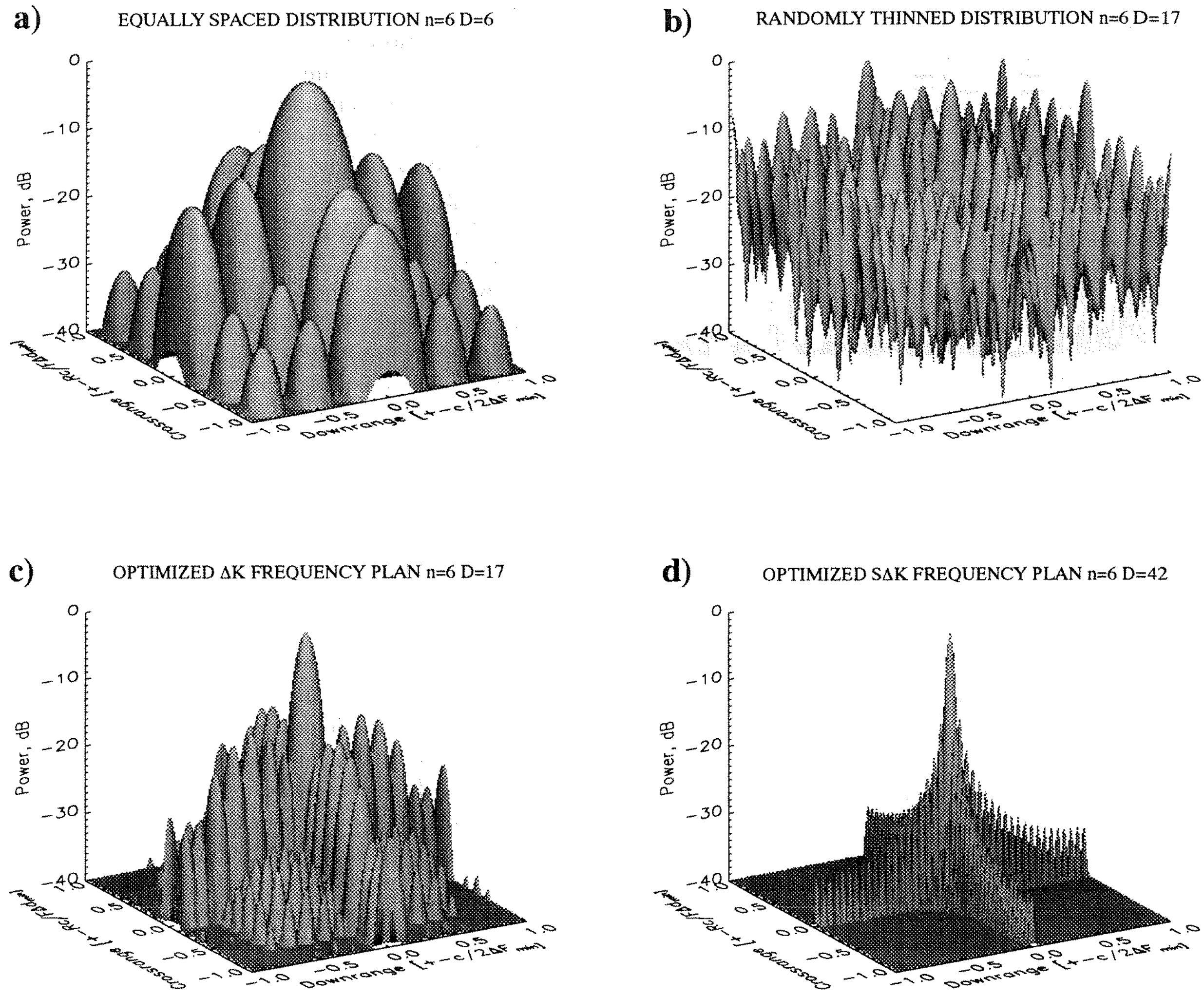

Figure 10. The signal-processing gain for the four different distribution plans considered: (a) six equally spaced K frequencies and antennas, (b) a randomly thinned six-K frequency and antenna distribution, (c) an optimized ΔK frequency and antenna plan, and (d) an $S\Delta K$ optimized frequency and antenna distribution showing the enhanced signal processing gain of the synthetic ΔK system.

Table 7. Theoretical Signal Processing Gain in the Range Profile Calculations Based on First-Level ΔK as Compared with Synthetic ΔK Processing

Number of K Channels	Number of ΔK Channels	ΔK Processing Gain, dB	Number of $S\Delta K$ Channels	$S\Delta K$ Processing Gain, dB	Signal Processing Gain Improvement, dB
3	3	9.5	3	9.5	0
4	6	15.6	12	21.6	6
5	10	20.0	30	29.5	9.5
6	15	23.5	65	36.3	12.8
7	21	26.4	97	39.7	13.3
8	28	28.9	121	41.7	12.8

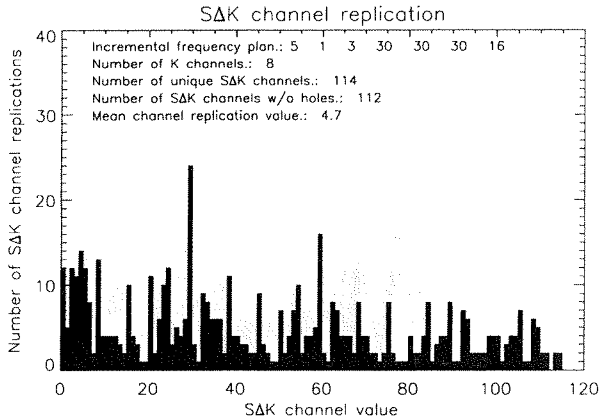

Figure 11. The channel replication based on an eight-K channel ΔK radar system using an incremental frequency plan of (5, 1, 3, 30, 30, 30, 16) resulting in 114 unique $S\Delta K$ channels, the first 112 without holes and with a mean channel replication factor of about 5. The accuracy of the phase estimate can be improved by $S\Delta K$ channel averaging.

c is the speed of light and Z is the range interval. Using a radar ($F = 10$ GHz) with three intermediate frequencies, $F_1 = 50$ MHz, $F_2 = 51.0$ MHz, and $F_3 = 51.999$ MHz, an $S\Delta K$ channel of $\Delta F = 1$ kHz is generated, and the range ambiguity interval becomes 150 km. While maintaining a range ambiguity interval of 150 km and an intermediate frequency bandwidth of typically 0.5 MHz, the maximum velocity measurement, v, becomes

$$v = \frac{\lambda}{2} f_{\mathrm{Doppler}} = \frac{c}{2F} f_{\mathrm{BW}} = \frac{3\times10^{8}}{2(10\times10^{9})}\,0.5\times10^{6} = 7500\ \mathrm{ms}^{-1} \tag{9}$$

4.4. Phase-Matching Applications

ΔK matching is obtained by measuring the phase relationship between the ΔK channels, and feature extraction applications involve computing the FFT in the time domain to derive the spectral content of the signals. Both these applications favor a large number of samples across the bandwidth, but the equal frequency spacing is not essential in this context. However, it appears that these applications can be realized easily and simultaneously using the synthetic ΔK method.

The seemingly unwanted ΔK channel replication is an added feature that can be utilized for target matching applications using frequency and phase diversity. Since the amplitude and phase deduced from identical ΔK channels can be derived using different carrier combinations, the accuracy of the ΔK phase relationship can be improved through averaging, and an improved inverse matched illumination filter, that is, the specific phase relationship between ΔK channels, can be constructed. The channel replication based on an eight K channel ΔK radar system is seen in Figure 11. From the eight initial K channels a total of 542 $S\Delta K$ channels are generated. Of these there are 114 unique unit-spaced ΔK channels, the first 112 without holes. The mean channel replication value is about 5, which means that the target phase match can be derived using an average of five phase estimates based on different combinations of the initial carrier signals, provided the radar is phase-calibrated for a single point source. Also, the identical ΔK channels can be coherently averaged to suppress the noise floor in the measurement accordingly.

4.5. Limitations in the Synthetic ΔK Technique

When multiplying two complex signals together, the clutter correlation present in the two data channels will add up and result in a degraded signal-to-clutter ratio (S/C). Multiplying the signals again will result in a further degradation of the S/C. One criterion for the $S\Delta K$ channel allocation technique to work is that there is a positive S/C prior to each multiplicative level. Some of the noise introduced in the multiplication can be compensated by bandwidth reduction using a digital low-pass filter after the multiplication. The bandwidth reduction factor, which is a function of the radar carrier frequency, F, and the ΔF frequency used ($F/\Delta F$), will typically vary between 10 and 1000. This will introduce a signal-processing gain of 20 to 60 dB and is one of the ΔK radar's main features. The multiplicative noise does not therefore seem to be a limiting factor.

Consider two reflectors, one of which has a cross section giving a 3-dB lower signal return than the other. At the first ΔK level the difference between the two reflectors will be 6 dB. Combining three K channels, the smaller reflector will be 9 dB down, and when combining four K channels, the difference will be 12 dB. This effect must be kept in mind when determining the number of successive ΔK levels to be implemented. This negative effect is, however, more than compensated for by the increased dynamic range in range profile calculations because of the increased number of samples across the radar bandwidth and the elimination of holes in the calculations.

The need for more signal-processing power is also evident because of the large number of new data

Table 8. Examples of Incremental Frequency Plans That Give Optimal $S\Delta K$ Channel Distribution by Maximizing the Number of ΔK Channels and/or Minimizing the Number of Holes in the Resulting Synthetic ΔK Frequency Plan

Number of K Channels	Incremental Frequency Plan	ΔK Channels	$S\Delta K$ Channels	$S\Delta K$ Channels Without Holes
4	(1, 8, 3)	6	12	12
5	(2, 9, 14, 15)	10	30	18
5	(4, 3, 15, 13)	10	29	22
6	(6, 13, 14, 15, 11)	15	48	40
6	(4, 12, 14, 15, 9)	15	46	42
7	(8, 15, 15, 14, 13, 12)	21	64	57
7	(8, 12, 13, 14, 30, 29)	21	85	22
8	(5, 1, 3, 30, 30, 30, 16)	28	114	112
8	(4, 1, 15, 29, 30, 30, 22)	28	121	91

channels generated. Manipulation of numbers using the synthetic ΔK method increases the number of computer operations by several orders of magnitude, depending on the number of initial K channels and the task at hand. While using only a few carriers to demonstrate the synthetic ΔK method in this paper, it would be more realistic, in a practical situation, to use a larger number of carriers so as to allow for a more complex target structure to be resolved.

5. Conclusions

This contribution presents a new method for signal generation and processing. It is based on a careful selection of the frequency plan tuning the frequency synthesizer circuitry to selected and preferred frequencies and specific signal-processing algorithms to synthesize the super ΔK channels.

The synthetic ΔK method [*Aarholt*, 1994] has the following advantages in that it (1) dramatically increases the number of ΔK channels available for object classification, (2) virtually eliminates the "holes" in the resulting ΔK frequency plan, (3) improves the dynamic range for inversion and range-profiling applications, (4) allows for extremely narrow synthetic filter implementation, eliminating common limitations in RF filter design, (5) simultaneously allows for extremely dense frequency channel distribution and high channel bandwidth allocation, (6) allows for ΔK channel phase averaging, (7) applies to antenna systems configuration to improve the antenna radiation pattern and directivity, and (8) enhances radar imaging by synthesizing frequency and antenna combinations. Table 8 lists some incremental frequency plans that are tuned for the maximum number of ΔK channels and/or minimum number of holes in the frequency plan, giving an optimal ΔK channel distribution for some radar applications.

Appendix

Synthetic ΔK Frequency Plans

The following section describes in detail how to allocate the available K channels of a radar system according to the $S\Delta K$ distribution theory together with examples of optimized K channel distributions which result in a combination of a minimum number of holes and a maximum number of $S\Delta K$ channels by minimizing the highest $S\Delta K$ channel values.

Radar Frequency Allocation Theory

The K channel values are obtained in the following way: (1) Set the first K channel frequency to zero. (2) The subsequent K channels are obtained by adding the elements in the incremental frequency table, that is, the values (1, 8, 3) in Table 8 give K channel values 0, 1, 9, and 12.

To calculate the absolute radar frequency values, two variables must be known: (1) the lowest intermediate frequency (F_1) of the radar system and (2) the bandwidth (BW) of the radar system. On the basis of the K channel distribution obtained above the absolute radar frequencies, F, are obtained using the following equation:

$$F_n = F_1 + \frac{(K\text{BW})}{K_{\max}} \quad \text{(A1)}$$

where K is the K channel frequency table.

Radar Frequency Calculations

Applied to real data, the $S\Delta K$ channels in Table A1 are based on the following calculations: incremental

Table A1. $S\Delta K$ Frequency Allocation

Channel Number	K Channel Combinations	$S\Delta K$ Channel Frequency, MHz
$S\Delta K_1$	$(K_1K_3^*)(K_2K_3^*)$	3.32917
$S\Delta K_2$	$(K_1K_3^*)(K_2K_4^*)$	6.65833
$S\Delta K_3$	$(K_1K_3^*)(K_1K_4^*)$	9.98750
$S\Delta K_4$	$(K_1K_4^*)(K_2K_3^*)$	13.3167
$S\Delta K_5$	$(K_2K_3^*)(K_3K_4^*)$	16.6458
$S\Delta K_6$	$(K_1K_3^*)(K_3K_4^*)$	19.9750
$S\Delta K_7$	$(K_1K_2^*)(K_2K_3^*)$	23.3042
$S\Delta K_8$	$(K_1K_2^*)(K_1K_3^*)$	26.6333
$S\Delta K_9$	$(K_1K_4^*)(K_3K_4^*)$	29.9625
$S\Delta K_{10}$	$(K_1K_2^*)(K_2K_4^*)$	33.2917
$S\Delta K_{11}$	$(K_1K_2^*)(K_1K_4^*)$	36.6208
$S\Delta K_{12}$	$(K_1K_4^*)$	39.9500

Table is based on the incremental frequency plan (1, 8, 3) giving unit increment and no holes in the frequency plan. The combinations of the original K channels forming the $S\Delta K$ channels are also shown.

Table A2. Synthetic ΔK Frequency Plans for Three to Eight K Channels Optimized for Minimum Number of Holes, Maximum Number of $S\Delta K$ Channels, and Minimum $S\Delta K$ Channel Value

Number of K Channels	Incremental Frequency Plan	Number of ΔK Channels	Number of $S\Delta K$ Channels	Number of $S\Delta K$ Channels Without Holes	$S\Delta K$ Channels
3	(1, 2)	3	3	3	1:3
4	(1, 8, 3)	6	12	12	1:12
5	(4, 3, 15, 13)	10	29	22	1:22 24:25 27:28 31:32 35
5	(2, 9, 14, 15)	10	30	18	1:18 20:21 23:27 29 31 36 38 40
6	(6, 13, 14, 15, 11)	15	48	40	1:40 42 44:48 53 59
6	(4, 12, 14, 15, 9)	17	46	42	1:42 45:46 50 54
7	(8, 15, 15, 14, 13, 12)	21	64	57	1:57 61:65 69 77
7	(8, 12, 13, 14, 30, 29)	21	85	22	1:22 24:57 59:69 71:74 76:79 81 84:86 90 92:94 98 106
8	(5, 1, 3, 30, 30, 30, 16)	28	114	112	1:112 114:115

Processing time for the eight-channel calculation can be in the order of a few weeks on a midrange UNIX workstation @ 1994.

Table A3. Synthetic ΔK Frequency Plans for Three to Eight K Channels Optimized for Maximum Number of $S\Delta K$ Channels and Minimum $S\Delta K$ Channel Value Stepping Through Unit Increments of 1–15 or 1–30 in the Incremental Frequency Plans

Number of K Channels	Incremental Frequency Plan	Number of ΔK Channels	Number of $S\Delta K$ Channels	Number of $S\Delta K$ Channels Without Holes	$S\Delta K$ Channels
3	(1, 2)	3	3	3	1:3
4	(1, 8, 3)	6	12	12	1:12
5	(2, 9, 14, 15) $n > 5000$ combinations	10	30	18	1:18 20:21 23:27 29 31 36 38 40
6	(4, 29, 28, 22, 19)	15	65	1	1 3:26 28:29 31:33 35:42 45:48 50:55 57 60:61 64:65 69:70 73:76 79:80 83 94 98 102
7	(8, 12, 29, 27, 25, 30)	21	97	33	1:33 35:57 59:64 66:76 79 81:82 84:86 89:91 93:94 96 98:99 101:104 106 111 115 119 123 131
8	(4, 1, 15, 29, 30, 30, 22)	28	121	91	1:91 93:112 115:116 121:123 125:127 130 131

frequency plan, (1, 8, 3); number of K channels, 4; K channel frequencies, 0, 1, 9, and 12; number of ΔK channels, 6; ΔK channel frequencies, 1, 3, 8, 9, 11, and 12; number of $S\Delta K$ channels, 12; $S\Delta K$ channel frequencies, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12; first radar frequency F_1 (MHz), 50.000; radar bandwidth BW (MHz), 39.950; radar frequencies (MHz), 50.0000, 53.3292, 79.9625, and 89.9500. Lists of various incremental frequency plans are shown in Tables A2 and A3.

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References

Aarholt, E., Multifrequency radar processing requirements and aircraft identification considerations. *NTNF-PFM Tech. Note TN-86/89*, 9 pp., Environ. Surveillance Technol. Prog., Lillestrom, Norway, 1989.

Aarholt, E., A system to generate super ΔK channels to enhance the classification of objects by means of selective radar frequency and antenna distribution plans, *Patent Appl. P943771*, The Norwegian Patent Office, Oslo, 1994.

Bracewell, R. N., Optimum spacings for radio telescopes with unfilled apertures, in *Progress in Scientific Radio, Report on the 15th General Assembly of U.R.S.I., Publ. 1468*, pp. 243–244, Natl. Acad. of Sci., Washington, D. C., 1966.

DiFranco, J. V., and W. L. Rubin, *Radar Detection*, 654 pp., Prentice-Hall, Englewood Cliffs, N.J., 1968.

Gjessing, D. T., *Adaptive Radar in Remote Sensing*, 154 pp., Ann Arbor Sci., Michigan, 1981.

Gjessing, D. T., *Target Adaptive Matched Illumination Radar, IEE Electromagn. Waves Ser.*, vol. 22, 172 pp., Peter Peregrinus, London, 1986.

Robinson, J. P., and A. J. Bernstein, A class of binary recurrent codes with limited error propagation, *IEEE Trans. Inf. Theory, IT-13* (1), 106–113, 1967.

E. Aarholt, Environmental Surveillance Technology Programme, PFM, Storgaten 6, P.O. Box 89, N-2001, Lillestrom, Norway. (e-mail: eldar.aarholt@pfm.no)

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